

# **Hybrid Functionals, van der Waals Corrections and ASE Interface to CP2K**

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# Part I: Hybrid Functionals

# What are hybrid functionals?

$$E_{xc}[\rho] = \alpha E_x^{\text{HFX}}[\{\psi_i\}] + (1 - \alpha) E_x^{\text{DFT}}[\rho] + E_c^{\text{DFT}}[\rho]$$

**Hybrid functionals:**

**mixing non-local Hartree-Fock exchange with  
local/semi-local DFT/GGA exchange**

# Why do we need hybrid DFT?

- Improved description of the thermochemistry (e.g. atomisation energy, heats of formation, etc) of molecular systems
- Improved description of the lattice constants, surface energies, ionisation potentials and band gaps of semiconductors and insulators

# Why do we need hybrid DFT?

|                    | Mae (G2) | Mae (G2-1) | Max ae (G2) | Max ae (G2-1) |
|--------------------|----------|------------|-------------|---------------|
| SVWN <sup>a</sup>  | 121.2    | 39.6       | 229         | 94            |
| LSD(SVWN5)         | 83.7     | 36.4       | 216         | 84            |
| PBE                | 17.1     | 8.6        | 52          | 26            |
| BLYP <sup>a</sup>  | 7.1      | 4.7        | 28          | 15            |
| VSXC <sup>b</sup>  | 2.7      | 2.5        | 12          | 8             |
| B3LYP <sup>a</sup> | 3.1      | 2.4        | 20          | 10            |
| PBE1PBE/PBE0       | 4.8      | 3.5        | 24          | 10            |

Mean absolute errors (Mae) for atomisation energies (kcal/mol)

# Why do we need hybrid DFT?

| Solid                | LSDA  | PBE   | TPSS  | HSE   |
|----------------------|-------|-------|-------|-------|
| ME <sup>a</sup>      | -1.14 | -1.13 | -0.98 | -0.17 |
| MAE <sup>b</sup>     | 1.14  | 1.13  | 0.98  | 0.26  |
| rms <sup>c</sup>     | 1.24  | 1.25  | 1.12  | 0.34  |
| Max (+) <sup>d</sup> | ...   | ...   | ...   | 0.32  |
| Max (-) <sup>e</sup> | -2.30 | -2.88 | -2.66 | -0.72 |

**Band gap error (eV) statistics for the SC/40 test set**

# Hybrid DFT Calculations with CP2K

- Total energy as a functional of the electron density

$$E[\rho] = T_s[\rho] + J[\rho] + E_{xc}[\rho] + \int v(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r}$$

- Exchange-correlation energy with a hybrid functional

$$E_{xc}[\rho] = \alpha E_x^{\text{HFX}}[\{\psi_i\}] + (1 - \alpha) E_x^{\text{DFT}}[\rho] + E_c^{\text{DFT}}[\rho]$$

# Hybrid DFT Calculations with CP2K

- Hartree-Fock exchange energy

$$E_x^{\text{HFX}}[P] = -\frac{1}{2} \sum_{\lambda\sigma\mu\nu} P^{\mu\sigma} P^{\nu\lambda} (\mu\nu|\lambda\sigma)$$

$$P^{\mu\nu} = \sum_i C^{ui} C^{vi} \Leftrightarrow P = CC^T$$

$$(\mu\nu|\lambda\sigma) = \iiint \phi_\mu(\mathbf{r}_1) \phi_\nu(\mathbf{r}_1) g(|\mathbf{r}_2 - \mathbf{r}_1|) \phi_\lambda(\mathbf{r}_2) \phi_\sigma(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$



**Four-centre two-electron integrals (ERIs):  $O(N^4)$  scaling**

# Interaction potential

$$(uv|\lambda\sigma) = \int \int \phi_\mu(\mathbf{r}_1) \phi_\nu(\mathbf{r}_1) g(|\mathbf{r}_2 - \mathbf{r}_1|) \phi_\lambda(\mathbf{r}_2) \phi_\sigma(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

- Standard Coulomb potential

$$g(r) = \frac{1}{r}$$

(used in B3LYP, PBE0, etc)

(see POTENTIAL\_TYPE keyword in &INTERACTION\_POTENTIAL subsection)

Guidon et al., J. Chem. Phys., 128, 214104 (2008)

# Interaction potential

$$(uv|\lambda\sigma) = \int \int \phi_\mu(\mathbf{r}_1) \phi_\nu(\mathbf{r}_1) g(|\mathbf{r}_2 - \mathbf{r}_1|) \phi_\lambda(\mathbf{r}_2) \phi_\sigma(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

- Range-separated Coulomb potential

**parameter**

$$\frac{1}{r} = \frac{\text{erfc}(\omega r)}{r} + \frac{\text{erf}(\omega r)}{r}$$

**short-range      long-range**  
(used in HSE06, etc)

(see POTENTIAL\_TYPE keyword in &INTERACTION\_POTENTIAL subsection)

Guidon et al., J. Chem. Phys., 128, 214104 (2008)

Heyd & Scuseria, J. Chem. Phys., 118, 8207 (2003)

# Interaction potential

$$(uv|\lambda\sigma) = \int \int \phi_\mu(\mathbf{r}_1) \phi_\nu(\mathbf{r}_1) g(|\mathbf{r}_2 - \mathbf{r}_1|) \phi_\lambda(\mathbf{r}_2) \phi_\sigma(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

- Truncated Coulomb potential **parameter**

$$g_{\text{TC}}(r_{12}) = \begin{cases} \frac{1}{r_{12}}, & r_{12} \leq R_c \\ 0, & r_{12} > R_c \end{cases}$$

(used in PBE0-TC-LRC, etc)

(see POTENTIAL\_TYPE keyword in &INTERACTION\_POTENTIAL subsection)

Guidon et al., J. Chem. Theory Comput., 5, 3010 (2008)

Spencer & Alavi, Phys. Rev. B, 77, 193110 (2008)

# Auxiliary Density Matrix Methods

$$E_{xc}[\rho] = \alpha E_x^{\text{HFX}}[\{\psi_i\}] + (1 - \alpha) E_x^{\text{DFT}}[\rho] + E_c^{\text{DFT}}[\rho]$$

$$E_x^{\text{HFX}}[P] = -\frac{1}{2} \sum_{\lambda\sigma\mu\nu} P^{\mu\sigma} P^{\nu\lambda} (\mu\nu|\lambda\sigma)$$

- Introducing auxiliary density matrix  $\hat{P} \approx P$

$$\begin{aligned} E_x^{\text{HFX}}[P] &= E_x^{\text{HFX}}[\hat{P}] + (E_x^{\text{HFX}}[P] - E_x^{\text{HFX}}[\hat{P}]) \\ &\approx E_x^{\text{HFX}}[\hat{P}] + (E_x^{\text{DFT}}[P] - E_x^{\text{DFT}}[\hat{P}]) \end{aligned}$$

approximation

**original**  
**density matrix**

**auxiliary**  
**density matrix**

$$P^{\mu\nu} = \sum_i C^{\mu i} C^{\nu i} \Leftrightarrow P = CC^T$$

$$\hat{P}^{\mu\nu} = \sum_i \hat{C}^{\mu i} \hat{C}^{\nu i} \Leftrightarrow \hat{C}\hat{C}^T$$

# ADMM basis sets

|   |   |                                  |   |  |  |  |  |  |  |  |  |   |   |  |  |  |  |  |  |
|---|---|----------------------------------|---|--|--|--|--|--|--|--|--|---|---|--|--|--|--|--|--|
| <b>1A</b>                                   | <b>H</b><br>1s <sup>1</sup>                 | <b>2A</b>                        |   |  |  |  |  |  |  |  |  |   |   |  | <b>8A</b>  |  |  |  |  |
| 1   |   |                                  | 3   | 4  |  |  |  |  |  |  |  |   |   | 2  | <b>He</b><br>1s <sup>2</sup>                                       |  |  |  |  |
| <b>Li</b><br>1s <sup>2</sup> s <sup>1</sup> | <b>Be</b><br>1s <sup>2</sup> s <sup>2</sup> |                                  |   |  |  |  |  |  |  |  |  |   |   | 10   | <b>Ne</b><br>1s <sup>2</sup> s <sup>2</sup> p <sup>6</sup>         |  |  |  |  |
| 11  | 12  | <b>Na</b><br>[Ne]3s <sup>1</sup> | <b>Mg</b><br>[Ne]3s <sup>2</sup>                | <b>3B</b>  | <b>4B</b>  | <b>5B</b>  | <b>6B</b>  | <b>7B</b>  | <b>8B</b>  |  | <b>1B</b>  | <b>2B</b>   | 5   | 6  | 7  | 8  | 9  | 10   |  |
| 19  | 20  | <b>K</b><br>[Ar]4s <sup>1</sup>  | <b>Ca</b><br>[Ar]4s <sup>2</sup>                | <b>Sc</b><br>[Ar]3d <sup>1</sup> s <sup>2</sup>    | <b>Ti</b><br>[Ar]3d <sup>2</sup> s <sup>2</sup>    | <b>V</b><br>[Ar]3d <sup>3</sup> s <sup>2</sup>     | <b>Cr</b><br>[Ar]3d <sup>5</sup> s <sup>1</sup>    | <b>Mn</b><br>[Ar]3d <sup>5</sup> s <sup>2</sup>    | <b>Fe</b><br>[Ar]3d <sup>6</sup> s <sup>2</sup>    | <b>Co</b><br>[Ar]3d <sup>7</sup> 4s <sup>2</sup>   | <b>Ni</b><br>[Ar]3d <sup>8</sup> 4s <sup>2</sup>   | <b>Cu</b><br>[Ar]3d <sup>10</sup> 4s <sup>1</sup>   | <b>Zn</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup>   | <b>Ga</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>1</sup>   | <b>Ge</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>2</sup>   | <b>As</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>3</sup>   | <b>Se</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>4</sup>   | <b>Br</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>5</sup>   | <b>Kr</b><br>[Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>6</sup>   |
| 37  | 38  | <b>Rb</b><br>[Kr]5s <sup>1</sup> | <b>Sr</b><br>[Kr]4d <sup>1</sup> s <sup>2</sup> | <b>Y</b><br>[Kr]4d <sup>2</sup> s <sup>2</sup>     | <b>Zr</b><br>[Kr]4d <sup>5</sup> s <sup>1</sup>    | <b>Nb</b><br>[Kr]4d <sup>5</sup> s <sup>1</sup>    | <b>Mo</b><br>[Kr]4d <sup>6</sup> s <sup>1</sup>    | <b>Tc</b><br>[Kr]4d <sup>6</sup> s <sup>2</sup>    | <b>Ru</b><br>[Kr]4d <sup>7</sup> s <sup>1</sup>    | <b>Rh</b><br>[Kr]4d <sup>8</sup> s <sup>1</sup>    | <b>Pd</b><br>[Kr]4d <sup>10</sup>                  | <b>Ag</b><br>[Kr]4d <sup>10</sup> 5s <sup>1</sup>   | <b>Cd</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup>   | <b>In</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>1</sup>   | <b>Sn</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>2</sup>   | <b>Sb</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>3</sup>   | <b>Te</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>4</sup>   | <b>I</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>5</sup>    | <b>Xe</b><br>[Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>6</sup>   |
| 55  | 56  | <b>Cs</b><br>[Xe]6s <sup>1</sup> | <b>Ba</b><br>[Xe]6s <sup>2</sup>                | 57-71  | 72   | 73   | 74   | 75   | 76   | 77   | 78   | 79  | 80  | 81   | 82   | 83   | 84   | 85   | 86   |
| 87  | 88  | <b>Fr</b><br>[Rn]7s <sup>1</sup> | 89-103  | 104  | 105  | 106  | 107  | 108  | 109  | 110  | 111  | 112   | 113   | 114  | 115  | 116  | 117  | 118  |  |
|   |   | <b>Ra</b><br>[Rn]7s <sup>2</sup> | Actinides                                       | [Rn]f <sup>1</sup> 6d <sup>2</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>3</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>4</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>5</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>6</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>7</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>8</sup> 7s <sup>1</sup> | [Rn]f <sup>1</sup> 6d <sup>9</sup> 7s <sup>1</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>1</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>1</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>2</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>3</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>4</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>5</sup> | [Rn]f <sup>1</sup> 6d <sup>10</sup> 7s <sup>2</sup> p <sup>6</sup> |

(see \$CP2K/cp2k/data/BASIS\_ADMM)

# ADMM basis sets

Choice of auxiliary basis set for ADMM

- FIT3: three Gaussian exponents for each valence orbital
- cFIT3: a contraction of FIT3
- pFIT3: FIT3 + polarization functions
- cpFIT3: cFIT3 + polarization functions
- aug-FIT3, aug-cFIT3, aug-pFIT3, aug-cpFIT3: augmented with a “diffuse” function

(see \$CP2K/cp2k/data/BASIS\_ADMM)

# ADMM basis sets

| 1A                              |                                 |                                     |  |  |  |  |  |  |  |  |  |  |  |  |  | 8A   |   |   |
|---------------------------------|---------------------------------|-------------------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|---|---|
| 1<br><b>H</b>                   | 2A                              |                                     |  |  |  |  |  |  |  |  |  |  |  |  |  | 2<br><b>He</b>   |   |   |
| 1s <sup>1</sup>                 | 1s <sup>2</sup>                 |                                     |  |  |  |  |  |  |  |  |  |  |  |  |  | 1s <sup>2</sup>  |   |   |
| 3<br><b>Li</b>                  | 4<br><b>Be</b>                  |                                     |  |  |  |  |  |  |  |  |  |  |  |  |  | 1s <sup>2</sup>  |   |   |
| 1s <sup>2</sup> 2s <sup>1</sup> | 1s <sup>2</sup> 2s <sup>2</sup> |                                     |  |  |  |  |  |  |  |  |  |  |  |  |  | 1s <sup>2</sup> 2s <sup>2</sup> p <sup>0</sup>                       |   |   |
| 11<br><b>Na</b>                 | 12<br><b>Mg</b>                 | [Ne]3s <sup>1</sup>                 | [Ne]3s <sup>2</sup>                                    | 3B   | 4B   | 5B   | 6B   | 7B   | 8B   |  | 1B   | 2B   | [Ne]3s <sup>2</sup> p <sup>1</sup>                                   | [Ne]3s <sup>2</sup> p <sup>2</sup>                                   | [Ne]3s <sup>2</sup> p <sup>3</sup>                                   | [Ne]3s <sup>2</sup> p <sup>4</sup>                                   | [Ne]3s <sup>2</sup> p <sup>5</sup>                  | [Ne]3s <sup>2</sup> p <sup>6</sup>                  |
| [Ar]3s <sup>1</sup>             | [Ar]3s <sup>2</sup>             | [Ar]3d <sup>1</sup> 4s <sup>2</sup> | [Ar]3d <sup>2</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>3</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>4</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>5</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>6</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>7</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>8</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>9</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>10</sup> 4s <sup>1</sup>                                 | [Ar]3d <sup>11</sup> 4s <sup>2</sup>                                 | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>1</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>2</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>3</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>4</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>5</sup> | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>6</sup> |
| 19<br><b>K</b>                  | 20<br><b>Ca</b>                 | 21<br><b>Sc</b>                     | 22<br><b>Ti</b>  | 23<br><b>V</b>   | 24<br><b>Cr</b>  | 25<br><b>Mn</b>  | 26<br><b>Fe</b>  | 27<br><b>Co</b>  | 28<br><b>Ni</b>  | 29<br><b>Cu</b>  | 30<br><b>Zn</b>  | 31<br><b>Ga</b>  | 32<br><b>Ge</b>  | 33<br><b>As</b>  | 34<br><b>Se</b>  | 35<br><b>Br</b>  | 36<br><b>Kr</b>                                     |   |
| [Ar]3s <sup>1</sup>             | [Ar]3s <sup>2</sup>             | [Ar]3d <sup>1</sup> 4s <sup>2</sup> | [Ar]3d <sup>2</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>3</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>4</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>5</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>6</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>7</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>8</sup> 4s <sup>1</sup>                    | [Ar]3d <sup>9</sup> 4s <sup>2</sup>                    | [Ar]3d <sup>10</sup> 4s <sup>1</sup>                                 | [Ar]3d <sup>11</sup> 4s <sup>2</sup>                                 | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>1</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>2</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>3</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>4</sup>                  | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>5</sup> | [Ar]3d <sup>10</sup> 4s <sup>2</sup> p <sup>6</sup> |
| 37<br><b>Rb</b>                 | 38<br><b>Sr</b>                 | 39<br><b>Y</b>                      | 40<br><b>Zr</b>  | 41<br><b>Nb</b>  | 42<br><b>Mo</b>  | 43<br><b>Tc</b>  | 44<br><b>Ru</b>  | 45<br><b>Rh</b>  | 46<br><b>Pd</b>  | 47<br><b>Ag</b>  | 48<br><b>Cd</b>  | 49<br><b>In</b>  | 50<br><b>Sn</b>  | 51<br><b>Sb</b>  | 52<br><b>Te</b>  | 53<br><b>I</b>   | 54<br><b>Xe</b>                                     |   |
| [Kr]5s <sup>1</sup>             | [Kr]5s <sup>2</sup>             | [Kr]4d <sup>1</sup> 5s <sup>2</sup> | [Kr]4d <sup>2</sup> 5s <sup>2</sup>                    | [Kr]4d <sup>4</sup> 5s <sup>1</sup>                    | [Kr]4d <sup>5</sup> 5s <sup>1</sup>                    | [Kr]4d <sup>6</sup> 5s <sup>2</sup>                    | [Kr]4d <sup>7</sup> 5s <sup>1</sup>                    | [Kr]4d <sup>8</sup> 5s <sup>1</sup>                    | [Kr]4d <sup>10</sup>                                   | [Kr]4d <sup>10</sup> 5s <sup>1</sup>                   | [Kr]4d <sup>10</sup> 5s <sup>2</sup>                                 | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>1</sup>                  | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>2</sup>                  | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>3</sup>                  | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>4</sup>                  | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>5</sup>                  | [Kr]4d <sup>10</sup> 5s <sup>2</sup> p <sup>6</sup> |   |
| 55<br><b>Cs</b>                 | 56<br><b>Ba</b>                 | 57-71<br>Lanthanides                | 72   | 73   | 74   | 75   | 76   | 77   | 78   | 79   | 80   | 81   | 82   | 83   | 84   | 85   | 86<br><b>Rn</b>                                     |   |
| [Xe]6s <sup>1</sup>             | [Xe]6s <sup>2</sup>             | Lanthanides                         | [Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup>   | [Xe]4f <sup>14</sup> 5d <sup>3</sup> 6s <sup>2</sup>   | [Xe]4f <sup>14</sup> 5d <sup>6</sup> 6s <sup>2</sup>   | [Xe]4f <sup>14</sup> 5d <sup>7</sup> 6s <sup>2</sup>   | [Xe]4f <sup>14</sup> 5d <sup>8</sup> 6s <sup>2</sup>   | [Xe]4f <sup>14</sup> 5d <sup>9</sup> 6s <sup>1</sup>   | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>1</sup>  | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup>  | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>1</sup> | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>2</sup> | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>3</sup> | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>4</sup> | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>5</sup> | [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> p <sup>6</sup> |   |   |
| 87<br><b>Fr</b>                 | 88<br><b>Ra</b>                 | 89-103<br>Actinides                 | 104<br><b>Rf</b>                                       | 105<br><b>Db</b>                                       | 106<br><b>Sg</b>                                       | 107<br><b>Bh</b>                                       | 108<br><b>Hs</b>                                       | 109<br><b>Mt</b>                                       | 110<br><b>Ds</b>                                       | 111<br><b>Rg</b>                                       | 112<br><b>Cn</b>   | 113<br><b>Uut</b>  | 114<br><b>Fl</b>   | 115<br><b>Uup</b>  | 116<br><b>Lv</b>   | 117<br><b>Uus</b>  | 118<br><b>Uuo</b>                                   |   |
| [Rn]7s <sup>1</sup>             | [Rn]7s <sup>2</sup>             | Actinides                           | [Rn]5f <sup>14</sup> 6d <sup>2</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>3</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>4</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>5</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>6</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>7</sup> 7s <sup>2</sup> * | [Rn]5f <sup>14</sup> 6d <sup>8</sup> 7s <sup>1</sup> * | [Rn]5f <sup>14</sup> 6d <sup>9</sup> 7s <sup>1</sup> * | [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>1</sup> *              | [Rn]5f <sup>14</sup> 6d <sup>11</sup> 7s <sup>2</sup> *              | [Rn]5f <sup>14</sup> 6d <sup>12</sup> 7s <sup>2</sup> *              | [Rn]5f <sup>14</sup> 6d <sup>13</sup> 7s <sup>2</sup> *              | [Rn]5f <sup>14</sup> 6d <sup>14</sup> 7s <sup>2</sup> *              | [Rn]5f <sup>14</sup> 6d <sup>15</sup> 7s <sup>2</sup> *              |   |   |

(see \$CP2K/cp2k/data/**BASIS\_ADMM\_MOLOPT**)

# ADMM basis sets

## Uncontracted basis sets

- FIT10:  $4s + 3p + 3d$
- FIT11:  $4s + 3p + 3d + 1f$  → recommended for solids
- FIT12:  $4s + 3p + 4d + 1f$
- FIT13:  $4s + 4p + 4d + 1f$

## Contracted basis sets (double- $\zeta$ quality)

- cFIT10 / cFIT11 / cFIT12 / cFIT13

Names of the ADMM basis sets for main group elements will differ slightly, and usually the first ADMM basis set will not contain polarisation function.

# Some general suggestions

- Always check the convergence of **CUTOFF**  
(see [http://www.cp2k.org/howto:converging\\_cutoff](http://www.cp2k.org/howto:converging_cutoff))
- Always check the convergence of properties (e.g. lattice parameters, band gaps) with respect to **supercell sizes**
- Always start from pre-converged GGA (e.g. PBE) wavefunction and geometry
- Always check the convergence of **primary and ADMM basis sets** (start from a small basis and gradually increase the size)
- ADMM has only been implemented for use with GPW

# Work Flow

- 1. Convergence test (primary basis, CUTOFF, supercell, etc)**
  - 2. GGA optimisation with selected primary basis**
  - 3. Name GGA wave function file for use with ADMM**
  - 4. Construct ADMM input with auxiliary basis (e.g. FIT3)**
  - 5. Run calculations and check convergence of ADMM basis**
- 

# Input Structure: ADMM

&DFT

```
...
BASIS_SET_FILE_NAME ./BASIS_MOLOPT      (files can be found in $CP2K/cp2k/data)
BASIS_SET_FILE_NAME ./BASIS_ADMM
WFN_RESTART_FILE_NAME ${project}-RESTART.wfn
&SCF
    SCF_GUESS RESTART
...
&END SCF
&AUXILIARY_DENSITY_MATRIX_METHOD
    METHOD BASIS_PROJECTION
    ADMM_PURIFICATION_METHOD MO_DIAG
&END AUXILIARY_DENSITY_MATRIX_METHOD
...
&XC
...
&END XC
&END DFT
&SUBSYS
    &KIND Si
        BASIS_SET DZVP-MOLOPT-SR-GTH
        AUX_FIT_BASIS_SET FIT3
        POTENTIAL GTH-PBE-q4
    &END KIND
&END SUBSYS
(see examples in $CP2K/cp2k/tests/QS/regtest-admm-1/2/3/4) 19
```

# Which functional to use?

- PBE0-TC-LRC

$$E_{xc}^{PBE0-TC-LRC} = \mathbf{a} E_x^{HF,TC}(\mathbf{R}_c) + \mathbf{a} E_x^{PBE,LRC}(\mathbf{R}_c) \\ + (1 - \mathbf{a}) E_x^{PBE} + E_c^{PBE}$$

Guidon et al., J. Chem. Theory Comput., 5, 3010 (2008)  
 Spencer & Alavi, Phys. Rev. B, 77, 193110 (2008)

- HSE06

$$E_{xc}^{HSE06} = \mathbf{a} E_x^{HF,SR}(\boldsymbol{\omega}) + (1 - \mathbf{a}) E_x^{PBE,SR}(\boldsymbol{\omega}) \\ + E_x^{PBE,LR}(\boldsymbol{\omega}) + E_c^{PBE}$$

Scuseria et al., J. Chem. Phys., 125, 224106 (2006)

“Empirical” parameters:  $\mathbf{a}$ ,  $R_c$ , and  $\boldsymbol{\omega}$

# Input Structure: PBE0 vs. HSE06

```

&XC
  &XC_FUNCTIONAL
    &PBE
      SCALE_X 0.75
      SCALE_C 1.0
    &END PBE
    &PBE_HOLE_T_C_LR
      CUTOFF_RADIUS 2.0
      SCALE_X 0.25
    &END PBE_HOLE_T_C_LR
  &END XC_FUNCTIONAL
  &HF
    &SCREENING
      EPS_SCHWARZ 1.0E-6
      SCREEN_ON_INITIAL_P FALSE
    &END SCREENING
    &INTERACTION_POTENTIAL
      POTENTIAL_TYPE TRUNCATED
      CUTOFF_RADIUS 2.0
      T_C_G_DATA ./t_c_g.dat
    &END INTERACTION_POTENTIAL
    &MEMORY
      MAX_MEMORY 2400
      EPS_STORAGE_SCALING 0.1
    &END MEMORY
    FRACTION 0.25
  &END HF
&END XC

```

## PBE0-TC-LRC

(**t\_c\_g.dat** can be found in \$CP2K/cp2k/data)

```

&XC
  &XC_FUNCTIONAL
    &PBE
      SCALE_X 0.0
      SCALE_C 1.0
    &END PBE
    &XWPBE
      SCALE_X -0.25
      SCALE_X0 1.0
      OMEGA 0.11
    &END XWPBE
  &END XC_FUNCTIONAL
  &HF
    &SCREENING
      EPS_SCHWARZ 1.0E-6
      SCREEN_ON_INITIAL_P FALSE
    &END SCREENING
    &INTERACTION_POTENTIAL
      POTENTIAL_TYPE SHORTRANGE
      OMEGA 0.11
    &END INTERACTION_POTENTIAL
    &MEMORY
      MAX_MEMORY 2400
      EPS_STORAGE_SCALING 0.1
    &END MEMORY
    FRACTION 0.25
  &END HF
&END XC

```

## HSE06

(see examples in \$CP2K/cp2k/tests/QS/regtest-admm-1/2/3/4)

## A few more remarks ...

- Libint library is used to calculate ERIs and their analytic derivatives, see \$CP2K/cp2k/INSTALL for more details on installation and linking with CP2K
- Hybrid DFT calculations of large systems are memory demanding, try to increase MAX\_MEMORY or run the job with more MPI processes
- For extremely large hybrid DFT calculations, try hybrid MPI/OpenMP binary (i.e. cp2k.psmp)

# Example: Diamond Band Gap

| method     | number of integrals | gap [eV] |
|------------|---------------------|----------|
| PBE (PBS)  |                     | 4.17     |
| PBE (ABS)  |                     | 4.37     |
| PBE0 (PBS) | 40 787 850 778 591  | 6.07     |
| PBE0 (ABS) | 23 561 509 497      | 6.25     |
| PBE0 ADMM1 | 24 816 897 009      | 6.03     |
| PBE0 ADMM2 | 24 795 460 638      | 6.02     |

3x3x3 supercell

# Part II: Van der Waals Corrections

# Where does van der Waals force come from?

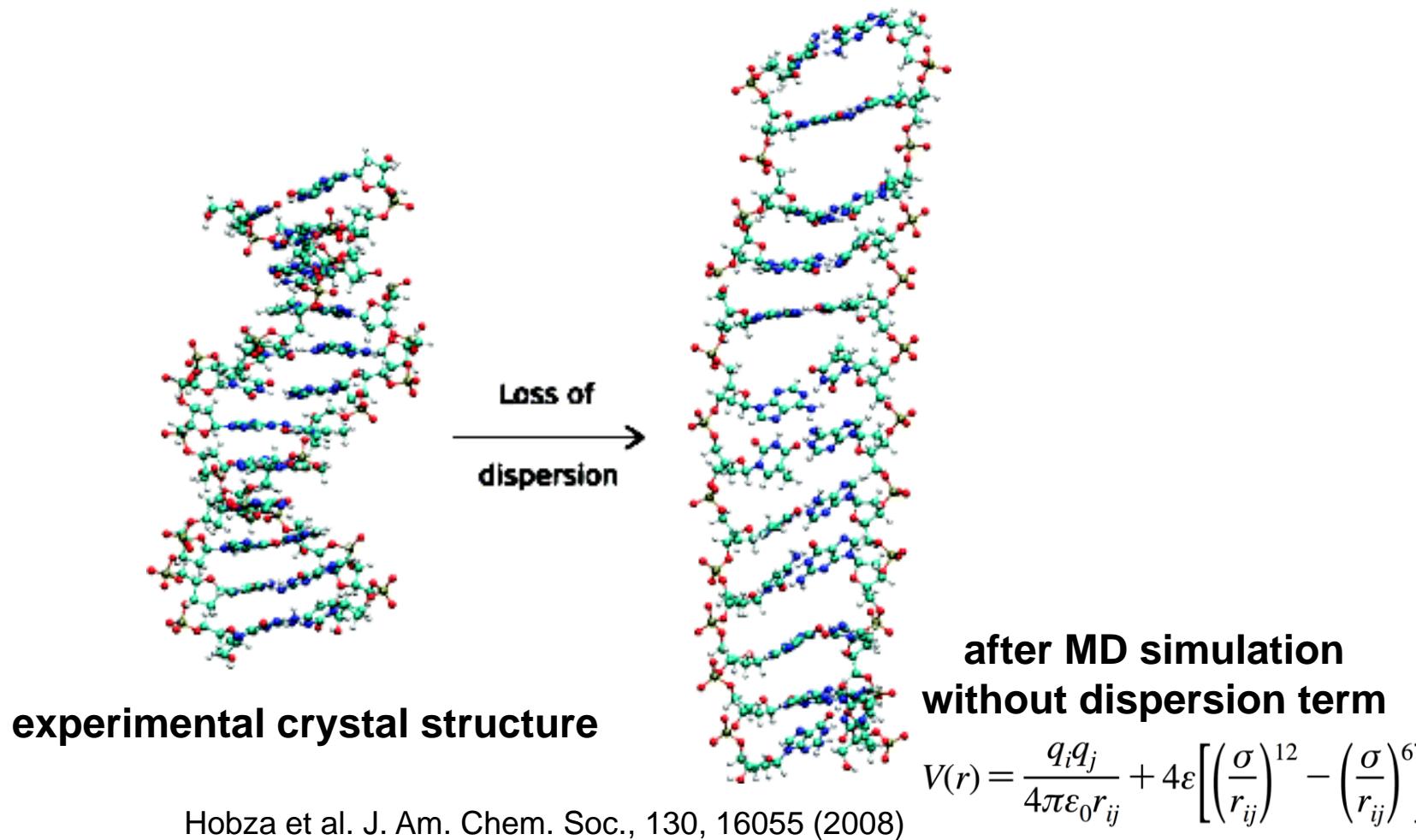


<http://www.youtube.com/watch?v=G1jGeeSWhXY>

**vdW force results from interaction of fluctuating charge distributions in one particle with those in an adjacent particle**

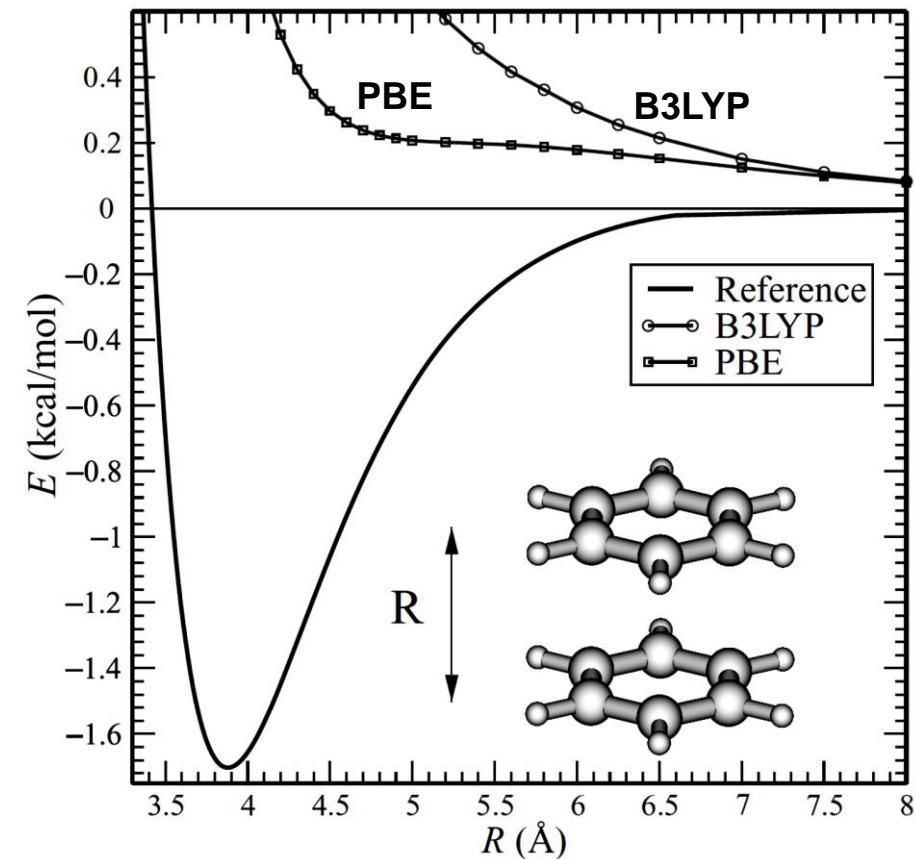
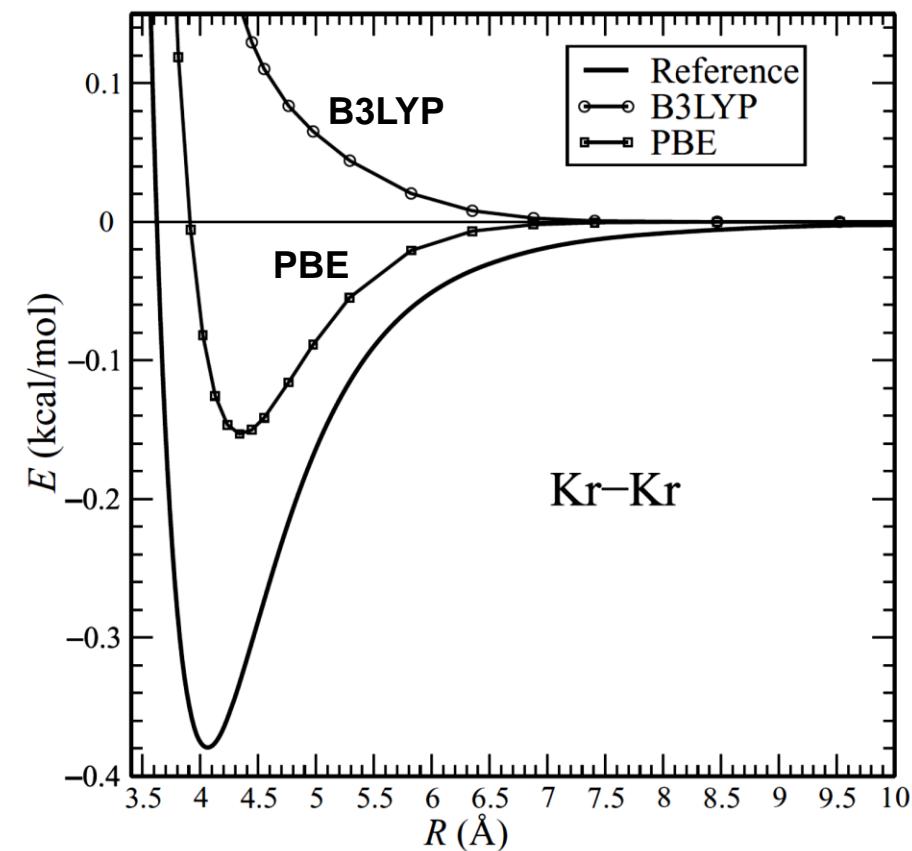
**(particle = atom, molecule, molecular fragment, etc)**

# Why van der Waals force is important?



**helical to ladder structural transition in DNA**

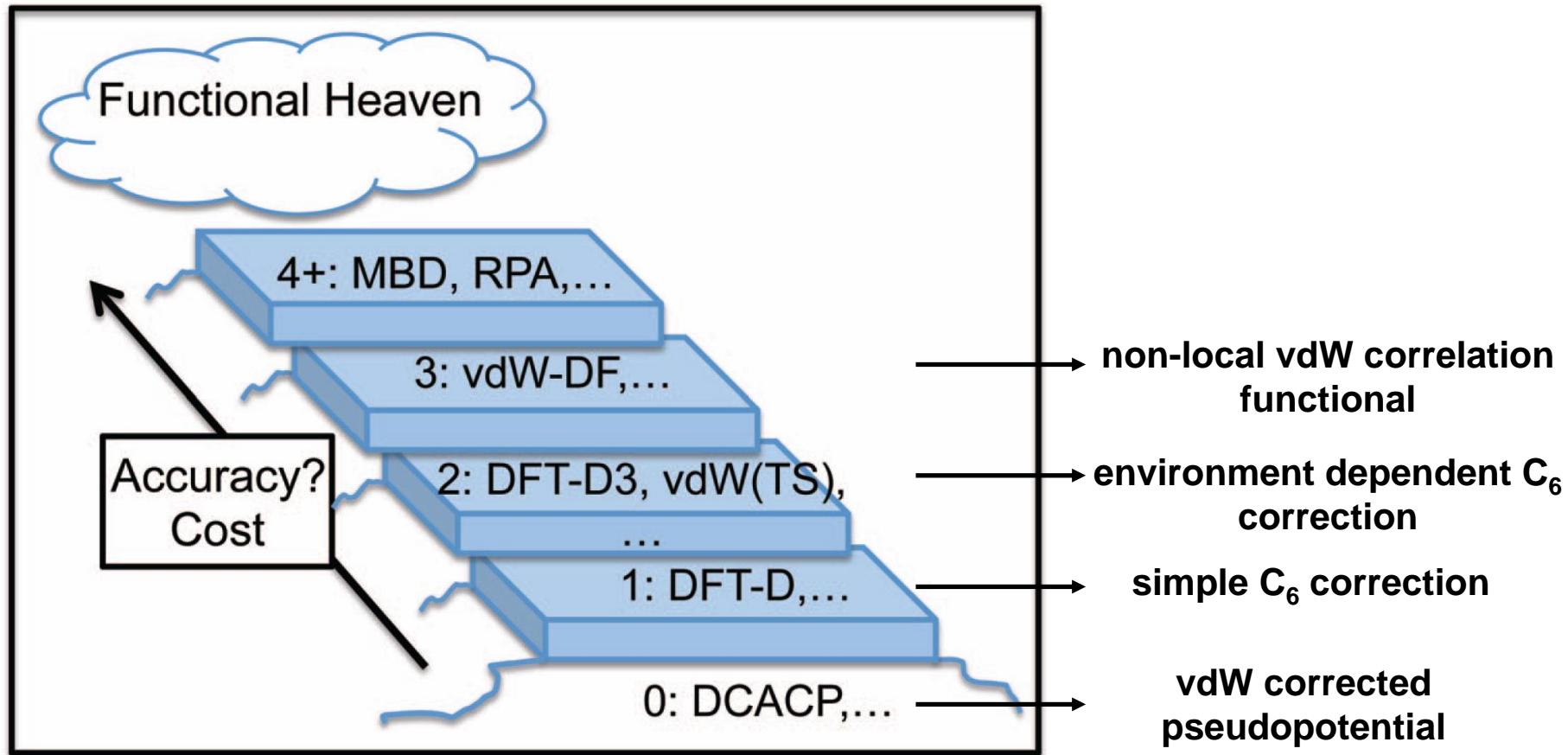
# Conventional DFT Fails for vdW Force



Reference: CCSD(T), see Grimme, WIREs Comput. Mol. Sci., 1, 211 (2011)

**Conventional DFT methods cannot provide  
correct  $-1/R^6$  dependence of vdW force on  $R$**

# Methods to include vdW force in Conventional DFT



Michaelides et al., J. Chem. Phys., 137, 120901 (2012)

**"stairway to heaven" for long range dispersion interactions**

# Simple C<sub>6</sub> Correction

$$E_{\text{disp}} = - \sum_{A,B} C_6^{\text{AB}} / r_{AB}^6$$

pairwise additive

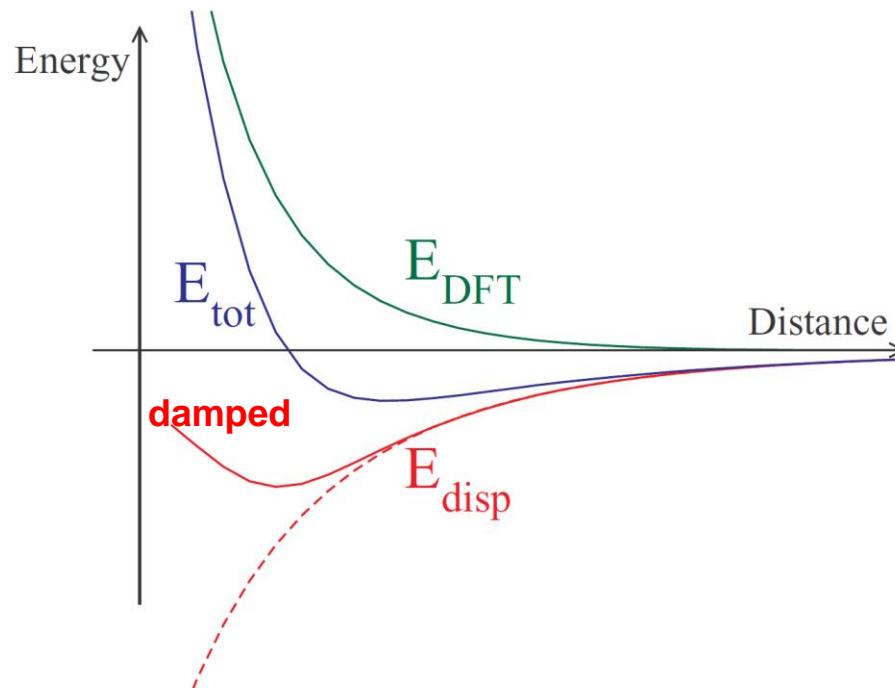
dispersion coefficients

$$E_{\text{tot}} = E_{\text{DFT}} + E_{\text{disp}}$$

↓

conventional DFT

# Damped C<sub>6</sub> Correction



$$E_{\text{disp}} = - \sum_{A,B} f(r_{AB}, A, B) C_6^{AB} / r_{AB}^6$$

Michaelides et al., J. Chem. Phys., 137, 120901 (2012)

## DFT+D2

$$C_6^{ij} = \sqrt{C_6^i C_6^j}$$

$$C_6^a = 0.05 N I_p^a \alpha^a$$

atomic ionization potentials

static polarizability

Grimme, J. Comput. Chem., 27, 1787 (2006)

# DFT+D2

```
&XC
```

```
  &XC_FUNCTIONAL PBE
```

```
  &END XC_FUNCTIONAL
```

```
  &vdW_POTENTIAL
```

```
    DISPERSION_FUNCTIONAL PAIR_POTENTIAL
```

```
    &PAIR_POTENTIAL
```

```
      TYPE DFTD2
```

```
      REFERENCE_FUNCTIONAL PBE
```

```
      SCALING ??? ← XC-dependent
```

```
      R_CUTOFF ??? ← cutoff radius to calculate Edisp
```

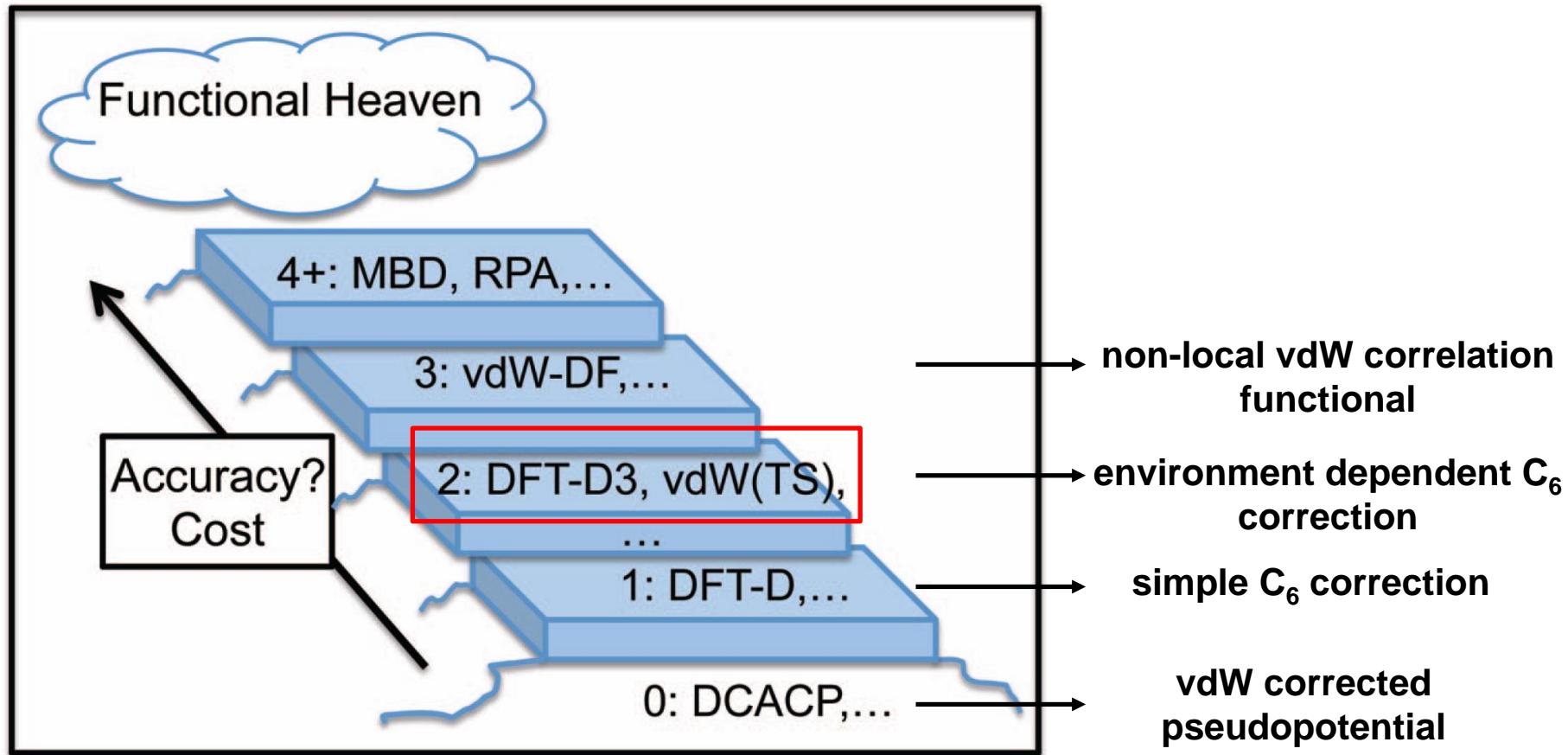
```
    &END PAIR_POTENTIAL
```

```
  &END vdW_POTENTIAL
```

```
&END XC
```

(see more examples in \$CP2K/cp2k/tests/QS/regtest-dft-vdw-corr-1)

# Methods to include vdW force in Conventional DFT



Michaelides et al., J. Chem. Phys., 137, 120901 (2012)

**“stairway to heaven” for long range dispersion interactions**

# Environment Dependent C<sub>6</sub> Correction

**DFT-D3:** pre-calculated C<sub>6</sub> coefficients for various pairs of elements in different hybridisation states

Grimme et al., J. Chem. Phys., 132, 154104 (2010)

**DFT-TS:** Hirshfeld partitioning of the molecular electron density, compare effective atomic volume with a free atom, and then scale the C<sub>6</sub> coefficients of a free-atom reference

Tkatchenko & Scheffler, Phys. Rev. Letts., 102, 073005 (2009)

# DFT+D3

$$C_6^{AB}(\text{CN}^A, \text{CN}^B) = \frac{Z}{W},$$

$$Z = \sum_i^{N_A} \sum_j^{N_B} C_{6,\text{ref}}^{AB}(\text{CN}_i^A, \text{CN}_j^B) L_{ij},$$

$$W = \sum_i^{N_A} \sum_j^{N_B} L_{ij},$$

$$L_{ij} = e^{-k_3[(\text{CN}^A - \text{CN}_i^A)^2 + (\text{CN}^B - \text{CN}_j^B)^2]},$$

$$\text{CN}^A = \sum_{B \neq A}^{N_{at}} \frac{1}{1 + e^{-k_1(k_2(R_{A,\text{cov}} + R_{B,\text{cov}})/r_{AB} - 1)}}$$

**coordination  
number**

## DFT+D3

$$E^{ABC} = \frac{C_9^{ABC}(3 \cos \theta_a \cos \theta_b \cos \theta_c + 1)}{(r_{AB} r_{BC} r_{CA})^3}$$

three-body term

$$C_9^{ABC} \approx -\sqrt{C_6^{AB} C_6^{AC} C_6^{BC}}$$

Grimme et al., J. Chem. Phys, 132, 154104 (2010)

# DFT+D3

&XC

&XC\_FUNCTIONAL PBE

&END XC\_FUNCTIONAL

&vdW\_POTENTIAL

DISPERSION\_FUNCTIONAL PAIR\_POTENTIAL

&PAIR\_POTENTIAL

TYPE DFTD3 ### or DFTD3(BJ)

← type of damping

CALCULATE\_C9\_TERM .TRUE.

← include three-body term

PARAMETER\_FILE\_NAME dftd3.dat

REFERENCE\_FUNCTIONAL PBE

D3\_SCALING ???

← XC and basis set dependent

R\_CUTOFF ???

← cutoff radius to calculate  $E_{\text{disp}}$

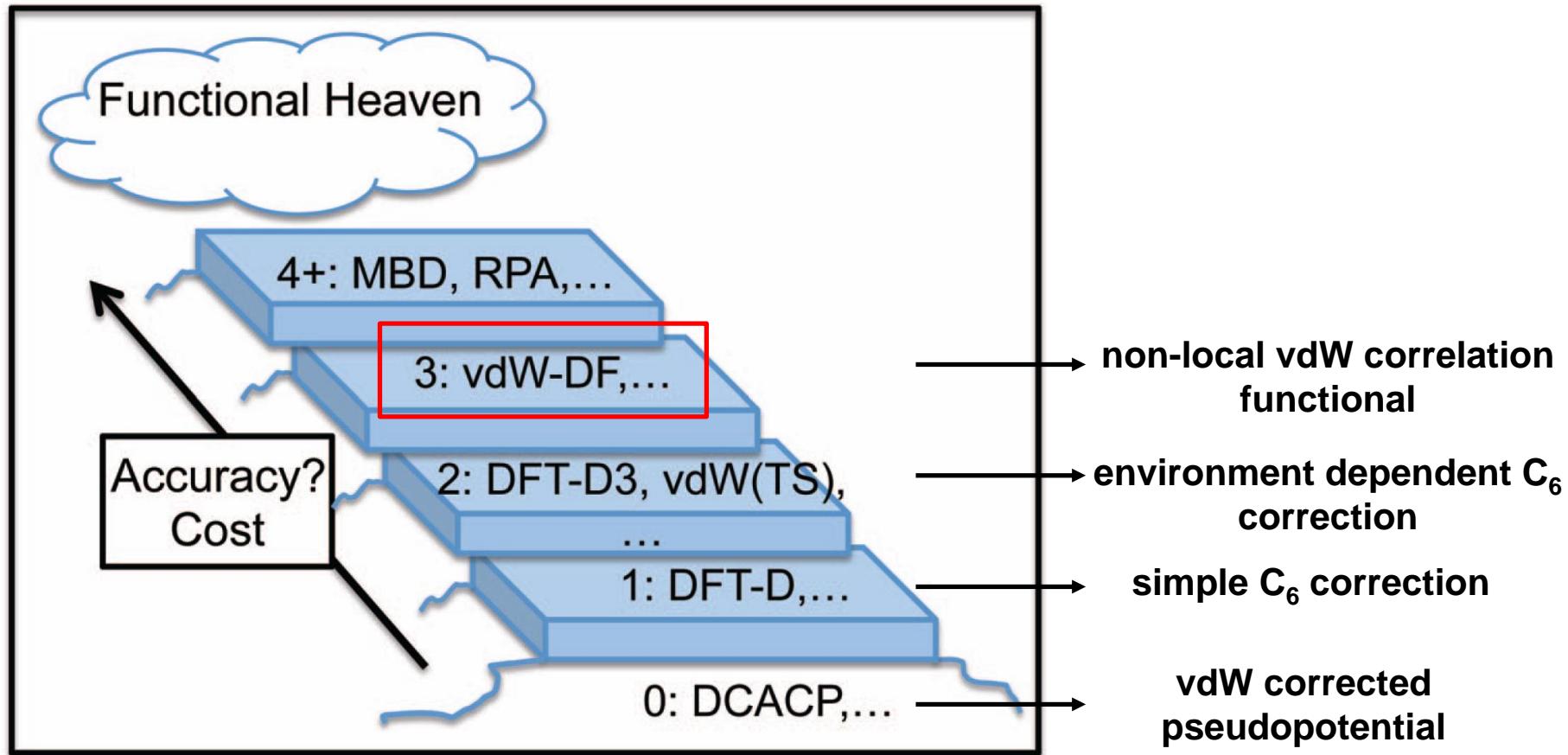
&END PAIR\_POTENTIAL

&END vdW\_POTENTIAL

&END XC

(see more examples in \$CP2K/cp2k/tests/QS/regtest-dft-vdw-corr-2;  
for additional scaling parameters optimised for **moderate** basis set  
calculations, see Supporting Information of the D3 paper)

# Methods to include vdW force in Conventional DFT



Michaelides et al., J. Chem. Phys., 137, 120901 (2012)

**"stairway to heaven" for long range dispersion interactions**

# Non-local vdW Correlation Functional

$$E_{\text{xc}} = E_{\text{x}}^{\text{GGA}} + E_{\text{c}}^{\text{LDA}} + E_{\text{c}}^{\text{nl}}$$

$$E_{\text{c}}^{\text{nl}} = \frac{1}{2} \int \int \rho(\mathbf{r}) \Phi(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') d^3 r d^3 r'$$

$$\Phi(\mathbf{r}, \mathbf{r}') = \Phi(\rho(\mathbf{r}), \rho(\mathbf{r}'), |\nabla \rho(\mathbf{r})|, |\nabla \rho(\mathbf{r}')|, |\mathbf{r} - \mathbf{r}'|)$$

Michaelides et al., J. Chem. Phys., 137, 120901 (2012)  
Hutter et al., J. Chem. Phys., 138, 204103 (2013)

# Non-local vdW Correlation Functional

| Functional            | Reference | Exchange         | Correlation                                |
|-----------------------|-----------|------------------|--|
| LDA                   | 1 and 33  | LDA              | LDA  |
| PBE                   | 5         | PBE              | PBE  |
| vdW-DF                | 19        | revPBE           | LDA+DRSLL                                  |
| vdW-DF2               | 26        | PW86R            | LDA+LMKLL                                  |
| C09 <sub>x</sub> -vdW | 29        | C09 <sub>x</sub> | LDA+DRSLL                                  |
| optB88-vdW            | 30        | optB88           | LDA+DRSLL                                  |
| RPBEc2/3+nl           | 31        | RPBE             | $\frac{1}{3}$ LDA+ $\frac{2}{3}$ PBE+DRSLL |
| rVV10                 | 34        | PW86R            | PBE+rVV10                                  |
| PBE-D3                | 18        | PBE              | PBE+D3                                     |
| revPBE-D3             | 18        | revPBE           | PBE+D3                                     |
| B97D-D3               | 18        | B97D             | B97D+D3                                    |

Hutter et al., J. Chem. Phys., 138, 204103 (2013)

# vdW-DF

```
&XC
  &XC_FUNCTIONAL
    &PBE
      PARAMETRIZATION revPBE
      SCALE_C 0.0
    &END PBE
    &VWN
    &END VWN
  &END XC_FUNCTIONAL
  &vdW_POTENTIAL
    DISPERSION_FUNCTIONAL NON_LOCAL
    &NON_LOCAL
      TYPE DRSLL ← type of non-local vdW correlation functional
      KERNEL_FILE_NAME vdW_kernel_table.dat
      CUTOFF ??? ← cutoff of FFT grid used to calculate  $E_c^{nl}$ 
    &END NON_LOCAL
  &END vdW_POTENTIAL
&END XC
```



revPBE exchange

LDA correlation

(see more examples in \$CP2K/cp2k/tests/QS/regtest-dft-vdw-corr-3)

# rVV10

```
&XC
  &XC_FUNCTIONAL
    &LIBXC
      FUNCTIONAL XC_GGA_X_RPW86 XC_GGA_C_PBE
    &END LIBXC
  &END XC_FUNCTIONAL
  &vdW_POTENTIAL
    DISPERSION_FUNCTIONAL NON_LOCAL
    &NON_LOCAL
      TYPE RVV10 ← type of non-local vdW correlation functional
      PARAMETERS 6.3 0.0093 ← parameters relevant to rVV10
      KERNEL_FILE_NAME rVV10_kernel_table.dat
      CUTOFF ??? ← cutoff of FFT grid used to calculate  $E_c^{nl}$ 
    &END NON_LOCAL
  &END vdW_POTENTIAL
&END XC
```

(see more examples in \$CP2K/cp2k/tests/QS/regtest-dft-vdw-corr-3)

## meta-GGA: M06-L

```
&XC
  &XC_FUNCTIONAL
    &LIBXC T
      FUNCTIONAL XC_MGGA_X_M06_L XC_MGGA_C_M06_L
    &END LIBXC
  &END XC_FUNCTIONAL
&END XC
```

Truhlar et al., J. Chem. Phys., 125, 194101 (2006)

- maybe combined with D3 correction
- analytical stress tensor has not been implemented, i.e.  
cannot be used for CELL\_OPT

# A few more remarks ...

- use higher planewave cutoff and tighter energy/force convergence criteria for meta-GGAs and functionals from Libxc
- to use functionals from Libxc, you will need to install Libxc and link CP2K with Libxc, see \$CP2K/cp2k/INSTALL for more details
- Basis set superposition error may “mimick” the missing dispersion interactions by DFT; check convergence on basis sets

# Part III: ASE interface

# How to use CP2K/ASE interface?

A sample CP2K/ASE job script (e.g. on ARCHER):

```
#!/bin/bash --login
#PBS -N CP2K-ASE
#PBS -l select=20
#PBS -l walltime=01:00:00
#PBS -A XXX-XXX → account number, replace with your own
#PBS -j oe

# modules that need to be loaded
module load cp2k
module load python-compute
module load pc-numpy
module load pc-scipy
module load pc-ase/3.11.0

# environment variables that need to be set
export ASE_CP2K_COMMAND="aprun -n 480 cp2k_shell.popt"
export CP2K_DATA_DIR=/work/y07/y07/cp2k/3.0.16521/data

# go to the job directory
cd $PBS_O_WORKDIR

# execute the CP2K-ASE job
python job.py
```

# How to use CP2K/ASE interface?

## A sample CP2K/ASE job input file:

```
#!/usr/bin/python
from __future__ import division, print_function
import os

from ase.structure import molecule
from ase.optimize import BFGS
from ase.calculators.cp2k import CP2K

def main():
    calc = CP2K(label='test_H2_GOPT')
    atoms = molecule('H2', calculator=calc)
    atoms.center(vacuum=2.0)

    # Run Geo-Opt
    gopt = BFGS(atoms, logfile=None)
    gopt.run(fmax=1e-6)

    # check distance
    dist = atoms.get_distance(0, 1)
    dist_ref = 0.7245595
    assert (dist - dist_ref) / dist_ref < 1e-7

    # check energy
    energy_ref = -30.7025616943
    energy = atoms.get_potential_energy()
    assert (energy - energy_ref) / energy_ref < 1e-10
    print('passed test "H2_GEO_OPT"')

main()
```

### Setting up the system:

```
calc = CP2K(label='sys')
sys = io.read('sys.xyz')
sys.set_cell([(8.0, 0.0, 0.0), (4.3, 7.0, 0.0), (0.0, 0.0, 21.4)],
             scale_atoms=False)
sys.set_pbc((True, True, True))
sys.set_calculator(calc)
```

# How to use CP2K/ASE interface?

## More advanced settings:

```
...
inp = """
&FORCE_EVAL
  METHOD Quickstep
  &DFT
    BASIS_SET_FILE_NAME BASIS_MOLOPT
    &MGRID
      CUTOFF 400
    &END MGRID
    &XC
      &XC_FUNCTIONAL LDA
      &END XC_FUNCTIONAL
    &END XC
    &POISSON
      PERIODIC NONE
      PSOLVER MT
    &END POISSON
  &END DFT
  &SUBSYS
    &KIND H
      BASIS_SET DZVP-MOLOPT-SR-GTH
      POTENTIAL GTH-LDA
    &END KIND
  &END SUBSYS
&END FORCE_EVAL
"""

```

```
...
# Basically, the entire CP2K input is passed in explicitly.
# Disable ASE's input generation by setting everything to None.
# For default settings, see $ASE/ase/calculators/cp2k.py.
# ASE should only add the CELL and the COORD section.
calc = CP2K(basis_set=None,
             basis_set_file=None,
             max_scf=None,
             cutoff=None,
             force_eval_method=None,
             potential_file=None,
             poisson_solver=None,
             pseudo_potential=None,
             stress_tensor=False,
             xc=None,
             label='test_H2_inp', inp=inp)
h2 = molecule('H2', calculator=calc)
h2.center(vacuum=2.0)
energy = h2.get_potential_energy()
energy_ref = -30.6989595886
diff = abs((energy - energy_ref) / energy_ref)
assert diff < 1e-10
print('passed test "H2_None"')

main()
```

# Further Reading

**Hybrid functionals in CP2K (by Sanliang Ling)**

[https://www.cp2k.org/\\_media/events:2015\\_cecam\\_tutorial:ling\\_hybrids.pdf](https://www.cp2k.org/_media/events:2015_cecam_tutorial:ling_hybrids.pdf)

**Van der Waals corrected DFT (by Sanliang Ling)**

[https://www.cp2k.org/\\_media/events:2015\\_cecam\\_tutorial:ling\\_vdw.pdf](https://www.cp2k.org/_media/events:2015_cecam_tutorial:ling_vdw.pdf)

and references therein.

**For Atomic Simulation Environment, see**

[https://wiki.fysik.dtu.dk/ase/\\_downloads/ase-talk.pdf](https://wiki.fysik.dtu.dk/ase/_downloads/ase-talk.pdf)